Comments on: ”Continuous initial observability of nonlinear delay parabolic equations” by Choi, Kwung and Park

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Abstract. It is shown that the assumptions of the main result of the cited paper can never be satisfied.

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In the paper [1] the problem of continuous initial observability is considered for the delayed infinite dimensional system

$$\begin{cases} x(t) &= S(t)\phi(0) + \int_0^t S(t-s)F(s,x_s)\mathrm{d}s, \quad t \in [0,T], \\ x(t) &= \phi(t), \quad t \in [-h,0], \\ y(t) &= \Pi x(t), \end{cases} \tag{1}$$

where $S(t)$ is a C_0-semigroup given on a Hilbert space, namely $X = L_2(\Omega)$, where Ω is a bounded domain in $\mathbb{R}^n$, Π is a bounded linear operator from X to the Hilbert space Y, F is a noon linear function. In order to investigate this problem, authors consider the linear non delayed part of the system:

$$\begin{cases} \dot{x}(t) &= Ax(t) \\ x(0) &= \phi(0) \\ y(t) &= \Pi x(t). \end{cases} \tag{2}$$

The main assumptions are: the semigroup $S(t)$ is analytical and compact (assumption H3) and the system (2) is initially continuously observable (assumption H1). Under those assumptions conditions of initial continuous observability of the system (1) are given.

We claim that the necessary condition of initial continuous observability of the system (2) is that the semigroup $S^*(t)$ is onto or that each operator $S(t)$ has a bounded inverse operator defined on $\operatorname{Im} S(t)$. If $S(t)$ is compact, then $S^*(t)$ is compact and cannot be onto in an infinite dimensional Hilbert space. If $S(t)$ is analytical then $\operatorname{Im} S^*(t) \subset \mathcal{D}(A^*)$ and then the operators $S^*(t)$ cannot be onto.

We need a precise definition of the initial observability because the definition given in [1] is not clear. We refer to [2, 3, 4].

Definition 1 *Let H be the operator defined by:*

$$H : X \to L_2(0, T; Y), \quad (Hx)(t) = \Pi x(t) = \Pi S(t)\phi(0).$$

The system (2) is said initially observable if $\ker H = \{0\}$*, that is the left inverse* H^{-1} *exists. It is said contiuously initially observable if* H^{-1} *is bounded.*

Note that the adjoint operator H^* is given by

$$H^* : L_2(0, T; Y) \to X, \quad H^* y(.) = \int_0^T S^*(t)\Pi^* y(t)\mathrm{d}t.$$

This gives that the system (2) is contiuously initially observable iff the dual system is exactly controllable.

It is well known that a necessary condition of exact controllability is that the corresponding semigroup is onto for all $t \geq 0$ (cf. for example [5, 6, 7]). That is a necessary condition of continuous initial observability is that the operators $S^*(t)$ are onto. Hence, the system (2) can never be continously initially observable if the semigroup $S(t)$ is analytical or compact in an infinite dimensional Hilbert space. The assumptions in the main result of [1] are contradictory.

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